AMENDMENTS TO THE SPECIFICATION

Please replace paragraph [0042] of the specification with amended paragraph [0042], below:

[0042] The factor short-term Momentum is measured over the last two days prior to execution. Momentum measures the price evolution of a stock within the last two trading days as a fraction of absolute price changes. Specifically,

~~$M = (Qn - Qo) / (\Sigma_{(n,i=1)} |Q_i - Q_{i-1}|$ Eq. (2) ,~~

$$M = (Q_n - Q_0) / \left(\sum_{i=1}^{n} |Q_i - Q_{i-1}| \right), \qquad \text{Eq. (2)}$$

where Q_0 and Q_n are the midpoints of the first and last valid primary quotes of the most recent two trading days and Q_i, $0 < i < n$, is the midpoint of the i'th valid primary quote occurring immediately prior to each valid primary trade of the most recent two trading days. Succinctly, a valid primary quote or trade is a quote or trade of a stock that occurred under regular market conditions on the stock's primary exchange.

Please replace paragraph [0046] of the specification with amended paragraph [0046], below:

[0046] The methodology of the present invention provides estimates for cost percentiles for any values of Size and Momentum from ~~[0,∞]~~ $[0, \infty)$ and [-1,1], respectively. Therefore, the methodology provides much more flexibility than actually needed when values of Size and Momentum are subdivided into different groups, and can be applied even if the choice of the ranges for Size and Momentum is different from the ones shown above.

Please replace paragraphs [0055]-[0058] of the specification with amended paragraphs [0055]-[0058], below:

[0055] Peer cost percentiles can be estimated for all benchmarks, clusterization types and possible choices of scenarios, assuming that at least one of the factors Size and Momentum has been selected. More precisely, the main result is to derive estimates of cost percentiles:

~~$X_i = Cost_{PercentileMarketCap=Y1,Side=Y2,Market=Y3,Size=Y4,Momentum=Y5}(i)$, Eq. (3)~~

$$X_i = \text{CostPercentile}_{MarketCap=y_1,Side=y_2,Market=y_3,Size=y_4,Momentum=y_5}(i), \qquad \text{Eq. (3)}$$

[0056] where ~~$y = (y1, y2, y3, y4, y5)$~~ $\underline{y} = (y_1, y_2, y_3, y_4, y_5)$ are arbitrary values for factors Market Capitalization, Side, Market, Size and Momentum, $i \in [0,100]$, and costs are measured relative to one of the six benchmarks discussed above.

[0057] Before estimating X_i in Eq. ~~eq.~~ (3), one must note that, first, while the factors Market Capitalization, Side and Market are discrete, Size and Momentum can have any values from ~~$[0,\infty]$~~ $[0,\infty)$ and [-1,1], respectively. Consequently, Eq. (3) consists of an infinite number of functions and thus, an infinite number of estimates have to be derived. Second, a pure empirical approach might not be practical in all cases. Subdividing factors Size and Momentum into different groups and computing the empirical distribution for each scenario may lead to inconsistency and instability. As a result, performance of costs realized from two very similar scenarios may be ranked very differently, which may be confusing for users. Third, it is preferred to have a methodology that provides robust estimates and that works for both clusterization types and all six benchmarks $C_{T\text{-}1}$, V_T, C_{T+1}, C_{T+20}, O_T and M_T. This requirement is important since various benchmarks (for instance, V_T and $C_{T\text{-}1}$) have very different properties.

[0058] In provisional application number 60/464,962, an ordinary least squares (OLS) methods method is described for providing estimates. The present invention does not focus on the mean or median only, but also provides estimates for the 25th, 40th, 60th and 75th costs percentiles in addition to the median. Instead of regressing all the cost percentiles in the comparison framework directly on the (total) trade size and momentum values, the present invention subdivides the comparison framework into different groups depending on the Momentum and Size of the executions (orders). Then, for each group, the 25th, 40th, 50th (median), 60th and 75th cost percentiles, are determined, as well as the equally weighted average values of momentum and (total) trade size.

Please replace paragraph [0065] of the specification with amended paragraph [0065], below:

[0065] The WLS1 approach is an enhancement of the OLS approach and comprises two steps: first, OLS regression is conducted and the residuals of the regression are determined; and second, the parameters are reestimated by weighting the observations with the inverse of their squared residuals. In order to avoid abnormal weighting, inverses of the squared residuals are truncated by the value $\cancel{(\Sigma^{n}_{i=1} e_i^2)^{-1}}$ $\underline{\left(\sum_{i=1}^{n} e_i^2\right)^{-1}}$.

Please replace paragraphs [0070]-[0072] of the specification with amended paragraphs [0070]-[0072], below:

[0070] Fig. 5 compares median cost estimates obtained by OLS, WLS1 and ~~LS2~~ WLS2 with empirical median costs. The ~~costs~~ dots denote the empirical medians. The solid line indicates the estimated median costs using the regression techniques WLS1. The two dotted lines show median cost estimates for OLS and WLS2. All estimates have been derived using regression ~~Eq. (3)~~ Eq. (4) for all executions in our data sample with $f(x) - x$ and $g(x) = 0$. Costs are measured relative to benchmark $C_{T\text{-}1}$. Empirical percentiles have been regressed on average size and momentum values, i.e. $f(x) = x$ and $g(x)$. The chart illustrates that all regression methods provide good estimates.

[0071] Fig. 6 compares median cost estimates obtained by OLS, WLS1 and WLS2 with empirical median costs. The dots denote the empirical medians. The solid line indicates the estimated median costs using the regression technique WLS1. The two dotted lines show median cost estimates for OLS and WLS2. All estimates have been derived using regression ~~equation (8)~~ Eq. (4) for all executions of Large cap stocks in ~~our~~ the data sample with $f(x) = x$ and $g(x) = 0$. Costs are measured relative to benchmark $C_{T\text{-}1}$. Instead of taking all executions into account, estimates have been derived for executions for Large cap stocks only. The functions f and g have been chosen linear again. Median cost estimates using OLS and WLS1 still do not differ considerably (WLS1 seems to yield slightly better results), however method WLS2 provides unreasonable estimates for large trade sizes.

[0072] Fig. 7 compares 25th-percentile estimates obtained by OLS, WLS1 and WLS2 with empirical 25th-percentiles of costs. The dots denote the empirical 25th-percentiles of costs. The solid line indicates the estimated 25th-percentile ~~24th percentile~~ using the regression technique WLS1. The two dotted lines show 25th-percentile estimates for OLS and WLS2. All estimates have been derived using regression ~~equation (8)~~ Eq. (4) for all executions in our data sample. f and g have been selected according to ~~equation (14)~~ Eq. (7). Costs are measured relative to benchmark M_T. Fig. 7 shows 25th-percentile estimates for executions and benchmark M_T using all data~~; f and g have been selected according to equation (7) below~~. By construction, WLS2 yields best results for small trade sizes, but underperforms the two other techniques when trade sizes increase.

Please replace paragraph [0080] of the specification with amended paragraph [0080], below:

[0080] The last constraint depends on the choice of the function g and on the type of benchmark. Typically, it is a technical condition on the parameters γ_i that ensures that (5) ~~(10)~~ doesn't happen.

Please replace paragraph [0083] of the specification with amended paragraph [0083], below:

[0083] $X_i = [[\alpha i]]\ \underline{\alpha_i} + \varepsilon_i,\ i = 25, 40, 50, 60 \text{ or } 75\underline{.}$ Eq. (6)[[.]]

Please replace paragraphs [0085]-[0087] of the specification with amended paragraphs [0085]-[0087], below:

[0085] $f(x) = [[f_i]]\underline{f_1}(f_2(x)) \text{ and } g(x) = |x|^{3/4},$ Eq. (7)

[0086] where

[0087] $[[f_i]]\underline{f_1}(x) = x^{1/10} \quad \text{and} \quad f_2(x) = \begin{cases} x^4 / 0.02^3, & x \le 0.02 \\ x, & x > 0.02 \end{cases}$ Eq. (8)

Please replace paragraphs [0092]-[0093] of the specification with amended paragraphs [0092]-[0093], below:

[0092] ~~Fig. 9~~ Fig. 10 shows estimated and realized cost percentiles versus trade sizes. The estimates are based on all executions that had momentum values within the range (-0.02, 0.02). All estimates have been derived using regression technique WLS1. f and g have been selected according to Eq. 7 ~~equation (14)~~. Costs are measured relative to benchmark M_T. In Figs. 8 and 10, the estimates are based on all executions with Momentum values within the range (-0.02,0.02).

[0093] ~~Fig. 10~~ Fig. 9 contains cost percentiles for all Large cap stocks and executions with Momentum values within the range (-0.02,0.02). As discussed above, the Figures show different behavior of cost percentiles for various benchmarks. Note that the scale on the y-axis varies considerably from benchmark to benchmark. Peer cost distributions for benchmark C_{T+20} are generally flat and heavy-tailed, and the form of the distribution does not change drastically as the trade size increases. This is different for the benchmarks V_T and M_T. In both cases, the standard deviations of peer cost distributions change considerably as trade sizes increase (for M_T it increases, for V_T it decreases).

Please replace paragraph [0099] of the specification with amended paragraph [0099], below:

[0099] Since actual transaction costs are extremely noisy and heavy-tailed, a robust method to build peer group cost distributions is required. The present invention provides a methodology that estimates peer cost percentiles for six different benchmarks, two different clusterization types and all possible choices of scenarios. In the present invention, trading costs can be grouped by the factors Type, Market Capitalization, Side, Market, Size and Short-term Momentum. While the first four factors have discrete values as input, it may be assumed that the factors Size and Momentum can have any values between ~~[0,∞]~~ $\underline{[0,\infty)}$ and [-1,1], respectively.

Please replace paragraph [00102] of the specification with amended paragraph [00102], below:

[00102] To measure performance of the two-step approach for an arbitrary scenario $\underline{y} = (y_1, y_2, y_3, y_4, y_5)$ for Market Capitalization, Side, Market, Size and Momentum one can compare the theoretical distributions with the corresponding empirical peer cost distributions (for y_4 and y_5 one can choose intervals $[y_4 - \Delta y_4, y_4 + \Delta y_4]$ and $[y_5 - \Delta y_5, y_5 + \Delta y_5]$). Comparing the theoretical with the empirical distributions provides an idea on how well the methodology works. Empirical studies performed by the present inventors have shown that in most cases estimated peer cost distributions are very close to the actual distributions. Percentile estimates of scenarios with very flat distributions ~~appei3r~~ appear to be less reliable. In particular, peer cost estimates for benchmark C_{T+20} might differ significantly from the empirical peer cost characteristics.